

The pion nucleon coupling constant and the Goldberger-Treiman Relation

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Abstract

The latest πN elastic scattering data are re-analysed to determine the coupling constant g_c of the charged pion, using the dispersion relation for the invariant amplitude $B^{(+)}$. Depending on the choice of data-base, values $g_c^2/4\pi = 13.80$ to 13.65 are obtained with errors of ± 0.12 . We re-examine the well known discrepancy with the Goldberger-Treiman relation. After allowing for the mass dependence of the pion decay constant f_π , a (2–3)% discrepancy is predicted, hence $g_c^2/4\pi = 13.74 \pm 0.10$ in the prior case.

The mass difference between charge states of $\Delta(1232)$ is $M^0 - M^{++} = 2.0 \pm 0.4$ MeV, close to twice the mass difference between neutron and proton. The difference in widths on resonance is $\Gamma^0 - \Gamma^{++} = 3.8 \pm 1.0$ MeV. One may account for a width difference of 4.5 MeV from phase space for decays and the extra channel $\Delta^0 \rightarrow \gamma n$.

1 Introduction

There have been long-running arguments over the magnitude of the pion-nucleon coupling constant. It appears that this argument can at last be resolved. The history will be reviewed briefly below, so as to set the present work in context. The dispute arises mainly from discrepancies in the normalisation of $d\sigma^\pm/d\Omega$ data and total cross sections in the region below the peak of the $\Delta(1232)$. More recent data clarify the experimental situation. The objective of the present paper is to make a fresh determination of the pion nucleon coupling constant $g_{\pi N\bar{N}}^2/4\pi$ with careful attention to (i) Coulomb barrier corrections, (ii) mass and width differences between Δ^{++} and Δ^0 . Hopefully, this re-analysis will settle at least some of the disagreements which have persisted for many years.

The Goldberger-Treiman relation [1] states that $f_\pi g_{\pi N\bar{N}} \simeq m_n g_A$, where m_n is the mean mass of the nucleon, 938.9 MeV, g_A the axial coupling constant of nucleon β decay, 1.267 ± 0.004 , and f_π the pion decay constant $f_\pi = 92.42 \pm 0.26$ MeV [2]. There is a well known discrepancy $\Delta = 2 - 3\%$ with this relation:

$$\Delta = 1 - \frac{m_n g_A}{f_\pi g_c}, \quad (1)$$

when one uses g_c , the coupling constant of charged pions to the proton; we shall distinguish later the coupling constant g_0 for neutral pions. We shall re-examine this discrepancy. The value of g_c is determined at the pole, $q^2 = 0$, whereas the experimental value of f_π is determined at $q^2 = m_\pi^2$. Today, the q^2 dependence of f_π is accurately under control from our understanding of Chiral Symmetry and how it is broken.

2 A brief historical review

Experiments at the CERN synchro-cyclotron in 1968-70 made precise measurements of πN scattering up to 290 MeV. Total cross sections for $\pi^\pm p$ were measured from 70 to 290 MeV, Carter et al. [3]; Bussey et al. reported $d\sigma^\pm/d\Omega$ from 88 to 292 MeV [4]; and the integrated cross section σ^0 for charge exchange from 90 to 290 MeV was measured by Bugg et al. [5]. A partial wave analysis of these data was made [6] including the effects of the Coulomb barrier and allowing for mass and width differences of the Δ ; these differences in mass and width were conspicuous in the total cross section

data. A value $g_c^2/4\pi = 14.28 \pm 0.16$ was found using the unsubtracted dispersion relation for the B^+ amplitude [7]. This value depended significantly on additional data then available at 310 MeV. These have been superseded by a series of experiments at PSI from 1975 to 1983 measuring differential cross sections and polarisations for elastic scattering and charge exchange. The effects of the new data were to reduce the width of the Δ slightly and hence reduce $g_c^2/4\pi$, though there was no fresh analysis at the time.

The 1973 analysis used Coulomb barrier corrections determined by solving a relativised Schrödinger equation [8]. A treatment of Coulomb effects based on dispersion relations was made by Tromborg et al. in 1977 [9].

Höhler and collaborators carried out extensive analyses of πN elastic scattering up to ~ 2 GeV using dispersion relations [10]. Comparisons are frequently made with the work of Koch and Pietarinen [11]. However, it should be realised that this analysis omitted the mass and width differences between Δ^{++} and Δ^0 . Some of the discrepancies subsequently reported between experiment and this analysis arise from this point.

Measurements were made of $d\sigma^\pm/d\Omega$ at TRIUMF by Brack et al. [12]. They reported substantially lower normalisation than the results of Bussey et al. in the mass range below 140 MeV. This discrepancy persists to this day.

De Swart and collaborators carried out a full analysis [13-16] of NN elastic scattering up to 350 MeV and reported considerably lower values of $g^2/4\pi$. Their 1993 values are $g_0^2/4\pi = 13.56 \pm 0.09$ for coupling to π^0 and $g_c^2/4\pi = 13.52 \pm 0.05$ for coupling to charged pions [16].

Arndt and collaborators re-analysed πN data using dispersion relation constraints and found $g_c^2/4\pi = 13.75 \pm 0.15$ [17]. However, this analysis omitted the total cross section data of Carter et al., and floated the normalisations of the Bussey et al. differential cross sections. It also treated the Coulomb barrier corrections in an approximate form. The main objective of the present analysis is to restore the missing data and the full treatment of the Coulomb barrier and see how much difference these make.

Recently, the Uppsala group has reported much higher values of $g_c^2/4\pi$ from measurements of np charge exchange differential cross sections: $g_c^2/4\pi = 14.52 \pm 0.26$ [18].

Meanwhile there have been extensive measurements at PSI and TRIUMF of differential cross sections and polarisations for $\pi^\pm p$ elastic scattering and charge exchange. All published values up to 2002 are included here using the SAID data base. The PSI measurement of the $\pi^- p$ scattering length via

pionic X-rays [19] is particularly important in providing an anchor point for the S -wave amplitude at threshold. It removes many of the uncertainties concerned with S -waves at low energies.

3 The $B^{(+)}$ dispersion relation

For πN scattering, the nucleon pole lies at $s = m_n^2$, almost midway between the physical regions for π^+p ($s > (m_p + m_\pi)^2$) and π^-p ($s < (m_p - m_\pi)^2$); here m_π is the mass of the charged pion. The determination of the $\pi N\bar{N}$ coupling constant is then a matter of interpolation between the physical regions for these two processes. This interpolation is more stable than the extrapolation which is required in NN analyses from the physical region to the pole at $t = m_\pi^2$. De Swart reports that, for the NN case, the coupling constant is determined mostly by low energy data; the determination must therefore come mostly from the u -channel pole below threshold at $s = 4m_n^2 - m_\pi^2$.

The $B^{(+)}$ dispersion relation contains a nucleon pole term and an integral over the imaginary part of the amplitude. In this particular combination of amplitudes, S -waves are suppressed by a large factor and the imaginary part of $B^{(+)}$ is dominated by P_{33} . The total cross sections and normalisations of differential cross sections therefore play an essential role in determining g_c^2 .

4 Ingredients in the Analysis

Our new analysis has been done using the SAID program [20] which constrains the data in the energy range up to 800 MeV using fixed t dispersion relations for $|t| = 0$ to 0.4 GeV². However, we do not impose the GMO sum rule. This relation relates the scattering length combination $(a_1 - a_3)$ to the nucleon coupling constant and an integral over total cross sections at all energies:

$$J = \frac{1}{4\pi^2} \int_{(m_p + m_\pi)^2}^{\infty} dk (\sigma_{tot}^- - \sigma_{tot}^+) / \nu,$$

where k is the lab momentum of the pion and ν its total lab energy. Bugg and Carter pointed out [21] that this integral is subject to a sizeable correction for the effect of the Coulomb barrier, which systematically enhances π^-p cross sections and suppresses π^+p . Furthermore, there is the danger that errors in cross sections at high energies bias the analysis of the region near threshold.

The formalism for Coulomb barrier corrections is described in the 1973 analysis of Carter et al. [6]. Here we try using as alternatives the corrections evaluated by both Tromborg et al. [9] and Bugg [8], in order to check the magnitude of any differences between them. There is a point here which deserves clarification. A superficial reading of these two papers suggests very different numerical values for corrections to π^-p . However, the two analyses adopt somewhat different approaches. The analysis of Tromborg et al. includes allowance in the numerical values of Coulomb barrier factors C_{ij} for the slightly different final-state momenta between $\pi^-p \rightarrow \pi^-p$ and $\pi^-p \rightarrow \pi^0n$. The partial wave analysis of Carter et al. instead allows specific phase space differences in these channels and accordingly introduces a small inelasticity into the P_{33} amplitude for π^-p scattering. When the analysis is run with the two alternative formalisms, results agree within one standard deviation for P_{33} and better than this for other partial waves. Numerical values of the Coulomb barrier calculations are available up to 500 MeV. At higher energies, extended-source Coulomb barrier factors supplied by Gibbs have been used [22].

There is a Coulomb term C_{13} which allows for explicit mixing between $I = 1/2$ and $3/2$; it arises from the fact that the Coulomb potential acts in the π^-p channel but not in π^0n . Including C_{13} into the analysis improves χ^2 significantly, by ~ 100 . More exactly, the fit has 21786 degrees of freedom; without C_{13} , $\chi^2 = 44850$ and including it $\chi^2 \rightarrow 44752$. It has the effect of increasing $g_c^2/4\pi$ by 0.03.

Our analysis also includes a mass difference between Δ^{++} and Δ^0 . This turns out to be essential.

There is an important detail concerning the total cross sections of Carter et al. There was an uncertainty of $\pm 0.25\%$ in beam momenta. On both wings of the Δ , cross sections vary rapidly with momentum; this introduces an error several times larger than the errors quoted for cross sections. We have added in quadrature to the experimental errors an error allowing for this uncertainty in beam momentum. This correction has been in the SAID data-base since 1980. For reference, values of these errors are given in Table 1.

Lab Energy (MeV)	σ_{tot}^+ (mb)	Lab Energy (MeV)	σ_{tot}^- (mb)
71.6	26.09 ± 0.62	76.7	15.80 ± 0.30
97.4	54.68 ± 0.66	96.0	23.12 ± 0.24
-	-	114.4	33.74 ± 0.34
118.9	96.25 ± 1.25	119.9	38.44 ± 0.47
120.4	101.04 ± 1.53	127.2	44.48 ± 0.46
136.0	141.19 ± 1.42	140.9	55.35 ± 0.43
138.7	148.60 ± 1.42	-	-
155.8	189.66 ± 1.08	159.6	67.90 ± 0.32
161.2	198.38 ± 1.09	164.7	70.26 ± 0.33
168.0	204.24 ± 0.85	172.6	70.74 ± 0.33
182.5	202.91 ± 1.03	184.6	69.76 ± 0.35
205.3	173.46 ± 1.27	208.9	59.43 ± 0.43
228.5	134.81 ± 1.32	232.6	47.06 ± 0.42
254.1	100.63 ± 0.97	255.4	38.46 ± 0.29
282.8	73.82 ± 0.91	290.1	29.97 ± 0.26

Table 1: Total cross sections with errors including uncertainties in beam momentum.

5 The essential discrepancy in data

The present status of the phase shift analysis may be summarised very simply. Most data sets contribute close to $\chi^2 = 1$ per point (when analysed without the dispersion relation constraint). There are few problems in fitting the *shapes* of differential cross sections or polarisations.

However, there is a well known discrepancy in data in the mass range below 145 MeV. On one side are the total cross sections σ_{tot}^+ and σ_{tot}^- of Carter et al. [3] and the normalisations of differential cross sections of Bussey et al. [4]. On the other side are the normalisations of differential cross sections of Brack et al. [12]. The shapes of differential cross sections from both Bussey et al. and Brack et al. may both be fitted adequately, but there are differences in normalisation.

There are faults here on both sides. The π^-p total cross section measurement of Carter et al. at the lowest energy 76.7 MeV is 7.5 standard deviations too high to fit the shape of the Δ . That can be seen in Fig. 2(b) of the 1973 phase shift analysis of Carter et al. [6]; the 76.7 MeV point lay

well above an effective range formula for P_{33} . The next point at 96.0 MeV lies suspiciously high by 3.8 standard deviations. We therefore reject both these π^-p total cross sections at 76.7 and 96 MeV. There is no difficulty for π^+p .

It is a matter of conjecture where the problem lies with the two π^-p points. The most likely explanation arises from $\pi \rightarrow \mu$ decays in the region of the target. At these low momenta, pions decay faster than they interact. There is a Jacobean peak in the decay angular distribution at a transverse momentum of 40 MeV/c. Energy loss in the full target is greater than in the empty target, increasing the decay rate. A correction is needed for this in the extrapolation to zero solid angle in the total cross section determination and may have been underestimated. The problem is worse for π^- than for π^+ because (a) elastic cross sections for π^- are lower than for π^+ by about a factor 9, (b) the beam size was somewhat larger for π^- than for π^+ .

Apart from this, there is one further discrepancy within CERN Data. At 263.7 MeV, the normalisation of Bussey et al. $d\sigma^-/d\Omega$ data is $\sim 5\%$ too low to agree with the difference between σ_{tot}^- and σ^0 . There is no problem with the total cross section data at this energy or with π^+p data. It is therefore desirable to free the normalisation of $d\sigma^-/d\Omega$ at this single energy.

Otherwise, normalisations of Carter et al. total cross sections are internally consistent with the integrated differential cross sections of Bussey et al. added to σ^0 . This provides a valuable check on resonance. There, partial waves other than P_{33} contribute $< 5\%$ of the π^+p total cross section and only 10% of the π^-p total cross section. The small partial waves are accurately determined from polarisation data via interference with P_{33} . On resonance, the P_{33} cross section is given by $8\pi/k^2$, leaving no freedom in the absolute normalisation. The data satisfy this check within experimental errors of typically 0.5%. Of course, it is still possible that normalisation errors develop at lower momenta. Incidentally, the data of Pedroni et al. [23] show 3 standard deviations disagreements on resonance with this check, despite their larger errors.

The relative normalisations of Brack et al. $d\sigma^\pm/d\Omega$ data are lower than those of CERN data by amounts up to $\sim 10\%$ in the mass range below 140 MeV.

This normalisation discrepancy affects primarily P_{33} and hence g_c^2 ; there are also small effects on the $\pi^\pm p$ scattering lengths. In order to estimate systematic errors arising from the choice of data set, we consider two extremes. In Fit I, the total cross sections of Carter et al. are removed and

the normalisations of Bussey et al are floated; data of Brack et al. are fitted according to published normalisations. In fit II, the procedure is reversed: normalisations of Brack et al. data and total cross sections of Pedroni et al. are floated; the published normalisations of Bussey et al. are retained and the total cross sections of Carter et al are fitted. In both cases, charge exchange cross sections of Bugg et al. are fitted, since these are the main source of information on P_{13} . In both cases, normalisations of differential cross sections of Frank et al.[24] and Bertin et al [25] are floated freely. The normalisation of the Frank et al. differential cross sections is low on average by 9% and the normalisation of Bertin et al. data is high on average by 12.4%.

6 Results

In Fit I (without Carter et al. and Bussey et al. normalisations), $g_c^2/4\pi$ optimises at 13.65. In fit II (floating Brack et al. data), $g_c^2/4\pi$ optimises at 13.80. In both cases, purely statistical errors are extremely small. If one uses the difference between the two results as a guide to systematic errors and attributes equal errors to both experiments, the systematic error on each is $\sim \pm 0.10$. Table 2 displays mean χ^2 per point for various data sets.

The normalisations of Brack et al. for $d\sigma^+/d\Omega$ in fit I give a large χ^2 of 4.17 per energy, even though the constraints on normalisation from the CERN data have been dropped. The problem is worst at 66.8 MeV where the fitted normalisation is 7% low, with a quoted error of 2.0%; at 86.8 MeV, it is low by 5% with a quoted error of 1.4%. What constrains these normalisations are the PSI X-ray data at threshold. In fit II, these discrepancies in normalisation increase to 11% and 7% respectively.

This discrepancy is apparent from the phase shift analysis of Fettes and Matsinos [26]. They analyse only data below 100 MeV and find a π^+p scattering length $a_3 = (0.077 \pm 0.003)m_\pi^{-1}$, compared with the value $0.0885^{+0.0010}_{-0.0021}m_\pi^{-1}$ from X-ray data.

The χ^2 for the normalisation of Brack et al. $d\sigma^-/d\Omega$ data is also quite high: 2.74 per energy. The problems in fit I are worst at 117.1 MeV, where the normalisation contributes a χ^2 of 15.85; at 66.8 and 87.5 MeV, it contributes 5.4 and 5.2 to χ^2 . At 45 MeV, χ^2 for the π^-p differential cross section is 8.30 per point. In Fit II, where CERN data are used with their published normalisations, the fitted normalisations of Brack et al. π^-p data are in the

Data set	χ^2 fit I	χ^2 fit II
Bugg et al. σ^0	0.52	0.87
Bussey et al, $d\sigma^+/d\Omega$	1.27	1.13
Bussey et al, $d\sigma^-/d\Omega$	1.49	1.93
normalisation, Brack et al. π^+	4.17	-
normalisation, Brack et al. π^-	2.74	-
Brack et al. $d\sigma^+/d\Omega$	1.76	1.02
Brack et al. $d\sigma^-/d\Omega$	2.50	1.91
Pedroni et al. σ_{tot}^+	1.51	-
Pedroni et al. σ_{tot}^-	2.30	-
Carter et al, σ_{tot}^+	-	1.38
Carter et al., σ_{tot}^-	-	1.16
Joram et al., $d\sigma^+/d\Omega$	3.39	3.95
Joram et al., $d\sigma^-/d\Omega$	2.08	2.26
Wiedner et al., $d\sigma^-/d\Omega$	2.65	2.60
Hauser et al., $d\sigma^0/d\Omega$	4.36	4.45
Gordeev et al., $d\sigma^+/d\Omega$	5.71	5.74
Gordeev et al., $d\sigma^-/d\Omega$	4.38	4.27

Table 2: Mean values of χ^2 per point in fits I and II.

range 0.86 to 0.95 for all energies from 117.1 MeV downwards.

It has often been remarked that σ_{tot}^- data of Pedroni et al. lie systematically lower than those of Carter et al. below the Δ resonance. On the other hand, their errors are large and overlap almost everywhere with the more precise data of Carter et al.

An escape route for the Brack et al. data is a possible violation of charge independence, allowing greater freedom. However, Meissner [27] estimates from Chiral Perturbation Theory that violations of charge independence in elastic scattering are unlikely to be above 1%. Meissner concludes that the 7% violation of charge independence proposed by Matsinos [28] appears unlikely.

The effect of this normalisation question is to make the Δ slightly narrower when the data of Brack et al. are used. That leads to a lower value of $g_c^2/4\pi$. Fit I, favouring the Brack et al. data leads to $g_c^2/4\pi = 13.65$; fit II, favouring CERN data, leads to $g_c^2/4\pi = 13.80$. However, this difference is now small because of the weight of other data. The latest analysis reported by Pavan at Menu 2001 [21] uses both CERN and Brack data and reports

$g_c^2/4\pi = 13.69 \pm 0.07$; this is slightly closer to fit I because of the larger number of points in the Brack et al. data.

Apart from these normalisation questions, the lower half of Table 2 shows a number of data sets with high χ^2 . The data of Joram et al. [29] and Weidner et al. [30] are mostly in the Coulomb interference region. The problem with the Joram et al. data lies in the shape of the differential cross sections; for Weidner et al. data the problem is that the normalisation is 10% high.

There are also high χ^2 for the data of Hauser et al. [31] and Goreev et al. [32]. These are not normalisation problems: it appears that errors have been underestimated. However, removing these data from the fit has little effect.

6.1 Errors from Coulomb barrier Corrections

There is some model dependence in Coulomb barrier corrections. Tromborg et al. include in their description of driving forces only the dominant nucleon exchange. Bugg [8] includes in addition σ , ρ and Δ exchanges. His results are $\sim 25\%$ lower in magnitude. Gashi et al. [33] evaluate corrections only up to 100 MeV. All work is subject to some systematic uncertainty in the σ interaction.

As an estimate of systematic errors from Coulomb barrier corrections, we take the difference between those of Tromborg et al. and Bugg. The former leads to the values of $g_c^2/4\pi$ quoted above; the latter gives values of $g_c^2/4\pi$ which are systematically lower by 0.06. Adding this error in quadrature to the systematic error arising from choice of data set, the overall systematic error is about ± 0.12 for $g_c^2/4\pi$.

6.2 Scattering Lengths

S -wave scattering lengths for fits I and II are shown in Table 3. Errors are systematic and are about $0.003m_\pi^{-1}$. Fit II is closer to the X-ray result: $2a_1 + a_3 = 0.2649 \pm 0.0024m_\pi^{-1}$. The TRIUMF data try to pull the π^-p scattering length to lower values. Both fits reproduce within errors the current algebra result that the symmetric combination of scattering lengths is zero.

	Fit I	Fit II
a_3	-0.0847	-0.0872
a_1	0.170	0.173
$2a_1 + a_3$	0.255	0.259
$a_1 - a_3$	0.255	0.260
$a_1 + 2a_3$	0.001	-0.001

Table 3: Scattering lengths in units of m_π^{-1} .

6.3 Mass and Width Differences for the Δ

Fitted masses and widths of Δ^0 and Δ^{++} and their differences are shown in Table 4 for both fits. All these values are after applying the correction for the Coulomb barrier. The masses are evaluated where the $\pi^\pm p$ phase shifts go through 90° . The mass difference is consistent with twice the mass difference between neutron and proton. We remark also that Pedroni et al [23] use deuterium data to find $\pi^- n$ cross sections; they give $M(\Delta^-) - M(\Delta^{++}) = 3.9$ MeV (no error quoted).

These differences in mass and width are visible by eye in the total cross sections of Carter et al.; an illustration of the difference in the Chew Low plot between $\pi^+ p$ and $\pi^- p$ is shown in Fig. 2(b) of Ref. [6]. However, the differences in mass and width are also required in fit I and are therefore clearly required also by differential cross section and polarisation data.

	Fit I	Fit II
$M(\Delta^{++})$	1231.45 ± 0.3 MeV	1231.0 ± 0.3 MeV
$M(\Delta^0)$	1233.6 ± 0.3 MeV	1232.85 ± 0.3 MeV
$M(\Delta^0) - M(\Delta^{++})$	1.86 ± 0.4 MeV	2.16 ± 0.4 MeV
$\Gamma(\Delta^{++})$	114.8 ± 0.9 MeV	115.0 ± 0.9 MeV
$\Gamma(\Delta^0)$	116.4 ± 0.9 MeV	118.3 ± 0.9 MeV
$\Gamma(\Delta^0) - \Gamma(\Delta^{++})$	1.6 ± 1.3	3.3 ± 1.3

Table 4: Masses and widths for the Δ from fits I and II.

Differences in width evaluated where the $\pi^\pm p$ phase shift goes through 90° are 1.16 ± 1.3 MeV for fit I and 3.3 ± 1.3 MeV for fit II. We now examine how to account for these differences in width. To first approximation, the

width may be parametrised as proportional to $k^3/(1+k^2R^2)$, where k is the decay momentum and R is an effective radius for the centrifugal barrier, taken to be 0.8 fm. Because k is larger in $\Delta^0 \rightarrow n\pi^0$ than for decay to π^-p , the width is larger. Allowing for the branching ratio 2 : 1 to these channels, one expects this phase space difference to make Γ^0 larger than Γ^{++} by 0.8 MeV for a given πp mass. However, in Table 4, Γ^0 is evaluated at a mass which is higher by 2 MeV than that for Γ^{++} . This contributes a further 2.6 MeV to Γ^0 . Thirdly, the Δ^0 has an extra width of 1.1 MeV for the γn channel. Adding these three effects, the expected width difference is 4.3 MeV. This estimate is close to Pilkuhn's estimate of 4.6 MeV [34]. These estimates are consistent with the observed difference in widths in fit II. We remark that the P_{33} phase shift is not sensitive to the total cross section near resonance. It is mostly sensitive near half-height of the resonance, when the P_{33} phase shift is close to 45 or 135°. Hence the width is sensitive to the formula used to parametrise the phase shift as a function of mass. Here, simple spline fits are used in the SAID program.

Kruglov [35] reports an independent Gatchina analysis of πN partial waves with the results:

$$M^0 = 1233.1 \pm 0.3 \text{ MeV} \quad (2)$$

$$M^{++} = 1230.5 \pm 0.2 \text{ MeV} \quad (3)$$

$$M^0 - M^{++} = 2.6 \pm 0.4 \text{ MeV} \quad (4)$$

$$\Gamma^0 - \Gamma^{++} = 5.1 \pm 1.0 \text{ MeV}. \quad (5)$$

From their original total cross section data, Carter et al. found $\Gamma^0 - \Gamma^{++} = 6.4 \pm 1.8 \text{ MeV}$. We consider all these values consistent in view of the different formulae used in different analyses.

7 Discrepancies with the Goldberger-Treiman relation

We return now to the Goldberger-Treiman relation, eqn. (1). Our general approach follows the ideas of chiral symmetry and its spontaneous breaking [36,37]. Our calculation of the discrepancy with the Goldberger-Treiman relation follows an algebraic approach proposed by Coon and Scadron [38].

The πN pole in the $B^{(+)}$ dispersion relation is at $s = m_n^2$, corresponding to $q^2 = 0$ for the mass of the pion. However, the pion decay constant f_π is

determined at $q^2 = m_\pi^2$. It is necessary to allow for possible q^2 dependence of f_π . The value of g_A is determined at $q^2 \simeq 0$ because of the low energy available in neutron beta decay.

The q^2 dependence of f_π may be obtained from a once-subtracted dispersion relation:

$$f_\pi(q^2) - f_\pi(0) = \frac{q^2}{\pi} \int_0^\infty \frac{dq'^2}{q'^2} \frac{Im f_\pi(q'^2)}{q'^2 - q^2}. \quad (6)$$

We suppose that the pion decays to two constituent quarks which in turn couple to W^\pm . For the quark loop,

$$Im f_\pi(q^2) = \frac{3g_{\pi q\bar{q}}}{2} \frac{4\hat{m}}{8\pi} \left(1 - \frac{4\hat{m}^2}{q^2}\right)^{1/2} F_\pi(q^2 - 4\hat{m}^2) \theta(q^2 - 4\hat{m}^2). \quad (7)$$

This follows from unitarity with the inclusion of a factor 3 for colour; $\hat{m} = (m_u + m_d)/2$ and m are constituent quark masses. At the quark level, the Golderger-Treiman relation is $f_\pi g_{\pi q\bar{q}} = \hat{m}$. Then, using a Taylor series expansion of the term $1 - q^2/q'^2$ in the denominator, eqn. (6) becomes

$$\frac{f_\pi(q^2) - f_\pi(0)}{f_\pi(0)} = \frac{m_\pi^2}{\pi} \int_{4\hat{m}^2}^\infty \frac{dq'^2}{q'^4} \frac{3g_{\pi q\bar{q}}^2}{4\pi} \left(1 - \frac{4\hat{m}^2}{q'^2}\right)^{1/2} \left(1 + \frac{q^2}{q'^2} + \dots\right) F_\pi(q'^2 - 4\hat{m}^2).$$

With $F_\pi = 1$, this may be evaluated analytically:

$$\Delta = \frac{f_\pi(m_\pi^2) - f_\pi(0)}{f_\pi(0)} = \frac{m_\pi^2}{8\pi^2 f_\pi^2} \left(1 + \frac{m_\pi^2}{10\hat{m}^2}\right). \quad (8)$$

The first term of the result is independent of \hat{m} . This term alone predicts a discrepancy $\Delta = 0.0295$; using $g_c = m_n g_A / [f_\pi(1 - \Delta)]$, we find $g_c^2/4\pi = 13.99$. The second term in (8) is very small and it is adequate to take $\hat{m} = m_n/3$. It leads to $\Delta = 0.0301$ and a prediction

$$g_c^2/4\pi = 14.01 \pm 0.10, \quad (9)$$

where the error arises from uncertainties in f_π and g_A .

For the form factor F_π , we use $F_\pi = \exp(-k^2 r_\pi^2/6)$, where k is the momentum in the $q\bar{q}$ loop. The pion charge radius is given [39] by $r_\pi^2 = 3/(4\pi^2 f_\pi^2) = (0.59 \text{ fm})^2$, in close agreement with the experimental value of 0.67 fm . Vector Dominance predicts $r_\pi = \sqrt{6}/m_\rho^2 = 0.63 \text{ fm}$. We use this intermediate

value. Folding in $F_\pi(q^2)$ numerically gives a factor 0.674 multiplying the right-hand side of eqn. (8). Finally, the prediction is

$$g_c^2/4\pi = 13.74 \pm 0.10; \quad (10)$$

this lies in the middle of the experimental values.

The value of $g_0^2/4\pi$ may also be evaluated by using in eqn. (8) the mass of the π^0 . The result is $g_0^2/4\pi = 13.70 \pm 0.10$; this is 1.5 standard deviations above the value of Stoks et al., 13.56 ± 0.09 .

A number of other determinations of the discrepancy Δ have been made by less direct methods [40-43] but with similar results.

8 Conclusions

From the $B^{(+)}$ unsubtracted dispersion relation, accurate values of $g_c^2/4\pi$ may be determined. The limitation at present is the systematic discrepancy between CERN and TRIUMF data below 145 MeV, although the addition of other data today reduces the effect of the discrepancy to quite a small value. The CERN data prefer $g_c^2/4\pi = 13.80 \pm 0.12$ and the TRIUMF data $g_c^2/4\pi = 13.65 \pm 0.12$. These errors include systematic uncertainties of ± 0.06 from uncertainties in the Coulomb barrier corrections. Normalisations of TRIUMF data are poorly fitted even when the normalisations of CERN data are floated.

The prediction from the Goldberger-Treiman relation, after allowing for the q^2 dependence of f_π from chiral symmetry, is $g_c^2/4\pi = 13.74 \pm 0.10$.

The mass difference between Δ^0 and Δ^{++} is consistent with twice the mass difference between neutron and proton. The observed difference in width is consistent with the effects of phase space and a 1.1 MeV width difference due to the extra channel $\Delta^0 \rightarrow \gamma n$.

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